Discrete Mathematics 118 (1993) 251–252
North-Holland

Note

The existence of a simple 3-(28, 5, 30) design

Yeow Meng Chee
Planning and Infrastructure Department, National Computer Board, 71 Science Park Drive, S0511, Singapore

Received 15 June 1990

A t-(v, k, λ) *design* is a pair $(X, \mathscr{B})$, where $\mathscr{B}$ is a collection of subsets of size k (called *blocks*) from a set X of cardinality v such that every t-element subset of X is contained in exactly λ blocks of $\mathscr{B}$. If the blocks in $\mathscr{B}$ are not repeated, the design is said to be *simple*. It is easy to show that the minimum value of λ for which a 3-(28, 5, λ) design can possibly exist is 30.

The existence of a 3-(28, 5, 30) design is known; Hanani, Hartman and Kramer constructed a 3-(28, 5, 30) design in [2]. However, their construction produces a

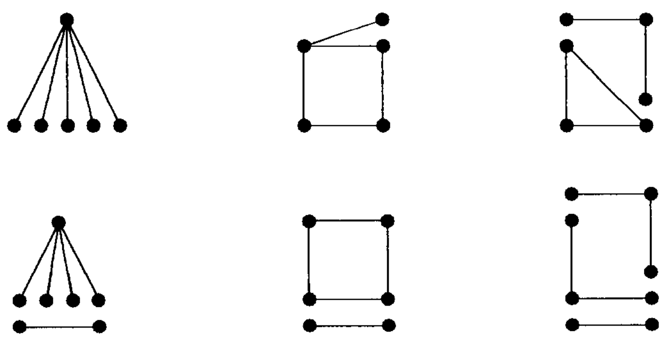

Fig. 1.

Correspondence to: Yeow Meng Chee, Planning and Infrastructure Department, National Computer Board, 71 Science Park Drive, S0511, Singapore.

0012-365X/93/$06.00

design with repeated blocks. The existence problem for simple 3-(28, 5, 30) designs is apparently not resolved (cf. [1]). In this note, we prove the existence of a simple 3-(28, 5, 30) design.

Let X be the set of $v=\binom{p}{2}$ labelled edges of the undirected complete graph K_p. A *graphical* t-(v, k, λ) design $(X, \mathscr{B})$ is one such that if $B \in \mathscr{B}$, then all subgraphs of K_p isomorphic to B are also in $\mathscr{B}$. In other words, $(X, \mathscr{B})$ has the symmetric group S_p as an automorphism group. We present a graphical 3-(28, 5, 30) design in Fig. 1.

Let X be the set of all 28 labelled edges of K_8. Take as blocks in $\mathscr{B}$ all the subgraphs of K_8 isomorphic to the six graphs shown in Fig. 1 (we omit isolated vertices for ease of presentation).

It is readily verified that $(X, \mathscr{B})$ is a 3-(28, 5, 30) design. Moreover, this design is simple.

References

[1] Y.M. Chee, C.J. Colbourn and D.L. Kreher, Simple t-designs with $v \leqslant 30$, Ars Combin. 29 (1990) 193–258.

[2] H. Hanani, A. Hartman and E.S. Kramer, On three-designs of small order, Discrete Math. 45 (1983) 75–97.